**Derivation of Root-finding Formulas for Some Special Fourth-order Equations of One Variable**

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For general quartic equations of one variable, there is the complex Ferrari formula. Here I will discuss some methods of solving equations that satisfy certain coefficient relationships.

For a certain quartic equation (The coefficient of the highest order term is changed to "1")

It may not be able to be transformed into the form of

Now suppose that something can be converted into this form, then we have

As for (1), let , then

As for (2), let , then

Equation (3) can be written as

It is the same equation as the original equation, so the coefficients should satisfy

By eliminating p and q by substitution, we get

Where

From (8) we get:

Which is a relationship about the original coefficient, which shows that only by satisfying this relationship can the hypothesis be established.

From (9), we get:

Substituting q, we get

Where n is still unknown, but it doesn’t matter, it can be eliminated later in the substitution process.

Substituting m and q into s and t, we get

Substituting s, t and p into x, we get

To sum up, if the fourth equation of one variable satisfies

Then its four solutions are shown as (11)

Example: Solving Equations

Test the coefficients, they satisfy

Substituting into the formula we get:

All are found to be the roots of the original equation.

This method of solving a quartic equation with specific coefficients is very simple. Of course, the four solutions in this example are all real numbers, but it doesn't matter if they are complex numbers. Now let's observe and the formula itself, found that there is no coefficient b in the formula, but in fact, .f